

MMA 863

**Intro to Analytic Modelling**

**Dr. Keith Rogers**

**MMA 863 Assignment 2**

**May 20th @ 11:59PM**

**Team Stirling**

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**Assignment Two**

**Question 1**

I run a pizza restaurant that is only open on Thursday, Friday and Saturday evenings from 5:30 to 9:30. Over that time, orders for pizza come into the restaurant randomly with a distribution that is approximately normal with mean 20 and standard deviation of 5 pizzas per hour. A pizza requires 1 kg of dough, 250 g of cheese and a mix of other ingredients depending on the order.

On Thursday AM I have 100 kg of dough on hand and am placing an order for more dough. The order will arrive the morning of the next day (Friday) on a truck from the bakery. This order is intended to last two weeks when the next order will arrive, also on a Friday morning.

**Question 1a:** How likely is it that I will run out of dough before the end of day Thursday?

**Answer 1a:**

If we consider the dough inventory alone, we can see from the ingredients of pizza that one order of pizza required 1kg of dough.

A batch of new dough supply would come the next morning, so the inventory of dough on Thursday AM only needs to satisfy the requirement of Thursday’s orders. Since orders for pizza come into the restaurant randomly with a distribution that is approximately normal with mean 20 and standard deviation of 5 pizzas per hour. The dough requirement for Thursday can be described by X ~ Normal(20\*4, 5\*sqrt(4)) = Normal(80,10) in kg.

The event of dough shortage would be X>100. P(X>100) = 1 – P(X<100) =1-NORM.DIST(100,80,5\*2,1) = 2.28%, so that is very unlikely.

However, this conclusion needs to be qualified with the assumption that this Thursday is just one of the ordinary operating days. If some extraordinary events occurred, say sports event, public interest in our pizza spiked, then such information needs to be incorporated into the modeling process.

Also we are estimating a quantity that is aggregate of the hourly dough requirements, the implicit assumption is that the hourly needs are independent of each other, if this assumption do not hold, the analysis needs to be re-worked.

**Question 1b:** Suppose that I actually have 200kg of dough on hand on the start of day Thursday. How much dough should I order to have no more than a 1% chance of stocking out by the time the next delivery arrives? (With 200kg of dough on hand, you can assume you make it through Thursday.)

**Answer 1b:**

A delivery of dough is supposed to last two weeks and this Thursday, or 7 operating days.

The dough requirement in kg can be described as X ~ Normal(20\*7\*4, 5 \* sqrt(7\*4)) = Normal(560, 26.46). to prevent the extreme situation of outage, we need to look at an threshold value Y, where P(X>Y) = 0.01.

We can work out Y =NORM.INV(1-0.01,560,26.46) = 621.55. since we already have 200kg on hand, the next order should be around 621.55 -200 = 421.55, or 422kg of resupply of dough.

**Question 1c:** Not many customers order double cheese. In fact, of the last 49 pizzas that were ordered, only 7 had double cheese. With this information, calculate the 95 % confidence interval on the proportion of customers who order double cheese.

**Answer 1c:**

This relates to the proportions, with the latest data on hand, we can have a point estimate the proportion as p = 7/49 = 14.28%. A side note, using the last orders for estimation does not mean these fit the random sampling criterion, so the aforementioned and below analysis needs to be taken with a grain of salt. It can be easily constructed that people may prefer cheesy pizza when it is getting late, and we would be over estimating.

For a pilot analysis, I shall suppress these doubts. Standard error of the proportions can be calculated as se = sqrt(p\*(1-p)/n) = 5.00% (n=49, p = 1/7). To create the confidence interval, with Z\_95% =-NORM.S.INV((1-95%)/2) = 1.96. then the interval would be [4.49%, 24.08%].

**Question 2**

CBC: I was listening to CBC radio as I drove in to the other day. They were speaking about results of a recent survey of 2000 from a population of people who owned five major appliances (fridge, oven, dishwasher, washer and dryer). Apparently only 300 of the people they contacted actually responded, but among them, 80% claimed that major appliances did not last as long as they used to, nor did they last as long as the 10 year average that appliance manufacturer’s association claimed.

Suppose that appliances actually last, on average 10 years, as the manufactures’ association claims. Assume that this has an approximately normal distribution with standard deviation of 3 years. What percentage of the survey population should have experienced a major appliance failure in fewer than 2 years?

**Answer 2:**

‘survey population should have experienced a major appliance failure in fewer than 2 years’

X < 2

Find P(X<2) given X~N(10,3)

Norm.Dist (2,10,3,TRUE) = 0.00383 =0.38%

**Therefore, 0.38% of the survey population should have experienced a major appliance failure in fewer than 2 years.**

**Question 3** Lots of parking lots

There are two parking lots at the college: A and B. Lot A is small with only 50 spots, B is much larger with 200 spots. If I get there after 8:15 AM, there is a 5% chance that A has space and a 10% chance that lot B has space. If lot A is has space then there is a 30% chance that lot B also has space.

**Question 3a:** If I show up at 8:15 AM, what is the probability I can get a parking spot?

**Answer 3a:**

What is known:

* P (A) = Probability of space in A = 0.05
* P (B) = Probability of space in B = 0.10
* P (B|A) = Probability of space in B if A has space = 0.3

P(A) = 0.05

P(~A) = 0.95

P(B|A) = 0.3

P(A AND B) = P(A) P(B|A) = 0.05\*0.3 = 0.015

Approach: Using Probability Tree

P(B|~A) = 0.085/0.95 = 0.089

P(~A AND B) = P(~A) P(B|~A) = 0.085

P(A AND ~B) = P(A) P(~B|A) = 0.05\*0.7 = 0.035

P(~B|A) = 0.7

\*Completing the tree using algebra:

P(~A AND ~B) = P(~A) P(~B|~A) = 0.865

P(~B|~A) = 0.911

P(B) = 0.1

P(B) = P(B and A) + P (B and ~A)

0.1 = 0.015 + P(B|~A)

P(B|~A) = 0.1 – 0.015 = 0.085

In mathematical terms:

P (A or B) = P(A) + P(B) – P(A and B) = 5% + 10% - 1.5% = 13.5%

**Therefore, the probability that I can get a parking spot is approximately 13.5%**

**Question 3b:** Assuming I drove all over the college and find the last remaining spot, what is the probability it is in Lot A?  
  
**Answer 3b:**

In mathematical terms, we will search for all probabilities in which P(A) is successful and P(B) is not.

**What we are trying to find in English terms:**

P(Parking available in A | Only 1 parking available remaining)

**Which can be expressed in Mathematical terms that:**  
= P[(A) |(A and ~B) OR (~A and B)]

= P(A and ~B) / [P(A and ~B) + P(~A and B)]

= 0.035/ (0.035+0.085)

= 0.035/0.12

= ~0.29

**Therefore, the probability that the last remaining spot is in 29%.**

**Question 4**

Some random problems – problems tend to be more difficult when they are removed from context. This is one reason why exams are perceived as more difficult than problems in textbooks. These are deliberately mixed around (as are real-world problems!) Provide details in your answer (e.g. parameters, number lines, pictures, functions) as appropriate to support your work.

**Question 4a:** Which is more likely: rolling 6 standard dice and getting either a 5 or 6 on at least 3 of them; or rolling 5 standard dice and getting a 4, 5 or 6 on at least 4 of them?

**Answer 4a:**

**It is more likely to roll a dice 6 times and get a 5 or 6 at least 3 times.**

Note: assumption that “rolling X standard dice” mean rolling 1 standard die X times

Is Poisson or Binomial a better fit? Binomial is a better fit: specific number of outcomes, with identical and independent experiments. Only two outcomes, success or failure.

Binomial

1. Success = roll 5 or 6

Number of success = 3 or more

Probability of success = 2/6 = 1/3

Number of trials = 6

1 - cumulative 2 successes

1. Success = roll 4 or 5 or 6

Number of success = 4 or more

Probability of success = 3/6 = 1/2

Number of trials = 5

1. - cumulative 3 successes
2. =1-BINOM.DIST(2,6,1/3,TRUE)= 0.319615912 = 32.0%
3. =1-BINOM.DIST(3,5,1/2,TRUE)= 0.1875 = 18.75%

**Question 4b:** During a thunderstorm, you see a lightning strike about once every 5 minutes. During a 20 min storm, what is the probability of their being between 5 and 7 strikes (inclusive)?

**Answer 4b:**

**During a 20 minute storm, there is 16.4% chance that there will be either 5, 6, or 7 lightning strikes.**

The Poisson distribution deals with situations where things happen randomly at a given rate over time or space

Poisson: (1) (lightning strike) every (5 min)

(4) (lightning strike) every (20 min)

probability 7 cuml – prob 5 cuml

**|--------------------4—THIS IS THE AREA----7--------------------|**

=POISSON.DIST(7,4,TRUE) - POISSON.DIST(4,4,TRUE)

= 0.948866384 - 0.628836935 = 0.320029449 = 32.0%

**Question 4c:** How many words would you have to pick, at random, from a dictionary, to know the proportion of words with the letter R in them within 1% with 95% confidence?

**Answer 4c: We would need to pick 98 words to be 95% confident of being within 1% of the actual proportion of words that include at least one R.**

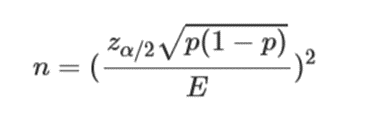
Solve for n

E = 1% = .01

CI = 95% = .95, alpha = 1 - .95 = .05

Z = =NORM.S.INV(1-0.05/2) = 1.959963985 = 1.96

We want a conservative estimate, so we use p =.5 (to maximize n)



n = [1.96√.5(1-.5)]/.01 = (1.96)(√.25)/.01 = (1.96)(.5)/.01 = 98

**Question 4d:** I need to have $100,000 in 5 years’ time. I am certain I can achieve 3% continuous growth if I invest in XYZ today. How much do I have to invest?

**Answer 4d: $86,071**

Using the exponential growth function:

P(future) = P(today) \* exp(rt)

100000=P\*e^(3%\*5)

P = $86,071

So we would say that you need to invest $86,071 today in order to get $100,000 5 years from now.